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Analytical solutions for constant-rate test in bounded confined aquifers with non-Darcian effect

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Abstract: This paper proposes a simplified analytical solution considering non-Darcian and wellbore storage effect to investigate the pumping flow in a confined aquifer with barrier and recharge boundaries. The mathematical modelling for the pumping-induced flow in aquifers with different boundaries is developed by employing image-well theory with the superposition principle, of which the non-Darcian effect is characterized by Izbash's equation. The solutions are derived by Boltzmann and dimensionless transformations. Then, the non-Darcian effect and wellbore storage are especially investigated according to the proposed solution. The results show that the aquifer boundaries have non-negligible effects on pumping, and ignoring the wellbore storage can lead to an over-estimation of the drawdown in the first 10 minutes of pumping. The higher the degree of non-Darcian, the smaller the drawdown.

Keywords: Non-Darcian effect; Bounded aquifer; Wellbore storage; Izbash's equation; Analytical solution

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Introduction

Mathematical modelling for groundwater flow induced by pumping is generally based on the assumption. It is suggested that the pumped aquifer is infinite and the drawdown-time curve cannot reach the real boundary (Mawlood and Mustafa, 2016). However, more and more field observations indicated that the aquifer system often encounters hydraulic barriers such as impervious rock, or recharge features such as rivers (Lu et al. 2015). In practice, the flow barrier or recharge features can be generally classified into the following boundary types: A single boundary, two boundaries at a right angle, two parallel boundaries, rectangular boundaries (Chan, 1976; Corapcioglu et al. 1983; Latinopoulos, 1984 and 1985), wedge-shaped boundaries (Holzbecher, 2005; Chen et al. 2009; Samani and Zarei-Doudeji, 2012; Samani and Sedghi, 2015; Kacimov et al. 2016a), triangle boundaries (Asadi-Aghbolaghi and Seyyedian, 2010), trapezoidal-shaped boundaries (Mahdavi and Seyyedian, 2014), reentrant angle boundaries (Lin et al. 2018) or meniscus-shaped domain (Kacimov et al. 2016b). Among them, aquifers with a single boundary and two boundaries are the most common bounded groundwater systems in practice. Aquifer with single boundary occurs mainly near the cliff, impermeable bedrock or stream (Kim, 2005; Zhang et al. 2007), and aquifer with two boundaries is usually located along the convex-bank of coast or river (Cherry, 2001), which is the main topic in this paper.

Studies on such boundary effects have started in the 1950s (Lin et al. 2018), which generally describe pumping response in the aquifers with different boundary conditions by using numerical models. The numerical solution is an approximate simulation method of an engineering situation, which mainly relies on mathematical methods called the finite element or the finite difference (Taigbenu, 2003; Loudyi et al. 2006; Kihm et al. 2007; Sergio and Serrano, 2013; Jafari et al. 2016). Theoretically, numerical modelling can accurately si-

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mulate pumping in a bounded aquifer. However, it is generally time-consuming and demands complicated preprocessing and computations (Younger and Paul, 2007). In comparison, the analytical approach is more convenient and more useful for hydrogeologists to investigate the physical characteristics of aquifers with simple settings. The image-well method is the most well-known tool for analytically modelling the well hydraulic problems in bounded aquifers (Lu et al. 2015). It allowed the user to remove the real boundary and site imaginary wells judiciously to take into account the boundary effects.

Based on literature review, it appears that the former analytical solutions for the bounded aquifers are physically proposed based on the assumption that the flow towards the well is under Darcian condition (Guadagnini et al. 2003; Chen et al. 2009; Mahdavi and Seyyedian, 2014; Samani and Sedghi, 2015). However, recent evidences suggest that the specific discharge could be nonlinearly related to the hydraulic gradient, defined as non-Darcian situation (Gavin, 2004; Wen et al. 2014; Xiong et al. 2016; Li et al. 2020; Hao et al. 2021). For instance, in the area nearby the pumping well, the hydraulic gradient is relatively higher and the Reynolds number is greater than 10, which can result in non-Darcian flow. Mathematically, Forchheimer's equation and Izbash's equation are proved to be the two most useful methods to describe such nonlinear relationship of wells for non-Darcian modelling (Mathias and Moutsopoulos, 2016; Liu et al. 2017; Xiao et al. 2019). Otherwise, in order to improve pumping efficiency, many engineers tend to use large diameter pumping wells, the radius can reach 2 m. Due to the influence of the wellbore radius and the wellbore storage, the drawdown behaviors of large-diameter wells are obviously different from the wells with infinitesimal radius (Wen et al. 2011; El-Hames, 2020). However, there have been few studies on the combined action of non-Darcian and wellbore storage effects on the drawdown simulation in the bounded aquifers so far.

As mentioned above, the image well method is equivalent to substituting the finite flow system into an aquifer system with several real and imaginary wells in the infinite areal extent. Such substitution is generally used to decompose the flow problem in the bounded aquifer to one of the adding effects of imaginary and real hydraulic systems in an infinite aquifer. It means that the flow behaviors in the bounded aquifer can be studied by using the superposition principle with analytical formulae in an infinite aquifer system. In the case of large diameter pumping well, the analytical solutions considering non-Darcian effect in an infinite aquifer have been derived since the 1980s (Sen, 1990; Wen et al. 2008a). However, it appears to the authors that the existing analytical solution is difficult to be used for the analytical modelling of drawdown simulation in bounded aquifers due to their computational complexity.

Hence, this paper firstly proposes a set of simplified analytical solutions considering non-Darcian flow with wellbore storage in confined aquifers. Due to its easy linearization process, the Izbash's equation can describe the nonlinear relationship for non-Darcian flow. By using the image-well theory, this paper further investigates the non-Darcian effect, represented by the quasi-hydraulic conductivity and non-Darcian index, and the wellbore storage on the drawdown behaviors in the bounded aquifer.

1 Solution derivation

1.1 Mathematical modelling in the confined aquifer

The schematic sketch of the pumping well with the wellbore storage in the confined aquifer is shown in Fig. 1. Q represents the pumping rate and b represents the thickness of the confined aquifer. The dotted line in Fig. 1 is the initial hydraulic head of the confined aquifer, and s is drawdown at a radius r from the pumping well. The assumptions are made as those for the this equation: (1) this aquifer is homogenous and horizontally isotopic, and it is hydrostatic before pumping; (2) the aquifer thickness is constant, and it extends infinitely horizontally; (3) the pumping well and observation wells are fully penetrating well.

In a confined aquifer, the following governing

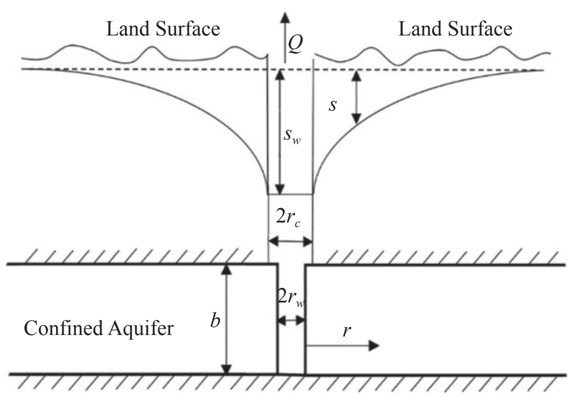

Fig. 1 Schematic sketch of constant rate test with wellbore storage

equation can be used to represent the pumping rate (Wen et al. 2008b).

$$\frac{\partial q}{\partial r}+\frac{q}{r}=\frac{S}{b}\frac{\partial s}{\partial t} \tag{1}$$

Where: b is the thickness of the aquifer, [L]; r is the distance from the observation well to the pumping well, [L]; t is the pumping time, [T]; $q(r,t)$ is the specific discharge, [LT^{-1}]; $s(r,t)$ is the drawdown, [L]; and S is the storativity.

The far away boundary condition is

$$s(r\to\infty,t)=0 \tag{2}$$

Considering the wellbore storage, the boundary representing the fully penetrating well is

$$\lim_{r\to r_w}2\pi brq(r,t)=-Q+\pi r_c^{\,2}\frac{\partial s_w(t)}{\partial t} \tag{3}$$

Where: Q is the constant pumping rate, [L^3T^{-1}]; r_w is the effective radius of pumping well screen, [L]; r_c is radius of pumping well casing, [L]; $s_w(t)$ is the drawdown inside the pumping well, [L].

The initial condition before the pumping is

$$s(r,0)=0 \tag{4}$$

Considering the non-Darcian effect, the non-linear relationship between the specific discharge and hydraulic gradient can been depicted by the Izbash's equation as

$$q=\left(K\frac{\partial s}{\partial r}\right)^{\frac{1}{n}} \tag{5}$$

Where: K is the quasi-hydraulic conductivity, [L^n/T^n]; n is the non-Darcian index; They are empirical constants in the Izbash's equation. $K^{\frac{1}{n}}$ is the hydraulic conductivity in the confined aquifer. n is a constant within the range of 1-2 representing the deviation degree from linearity. When n = 1, K is the hydraulic conductivity and the flow becomes Darcian.

1.2 Dimensionless transformation

The dimensionless transformation relationships are listed in Table 1, and the detailed clarification associated is in Supporting Information A. Based on Table 1, Equation (1) becomes dimensionless as

$$\frac{\partial q_D}{\partial r_D}+\frac{q_D}{r_D}=\frac{\partial s_D}{\partial t_D} \tag{6}$$

And the boundary conditions become

$$s_D(\infty,t_D)=0 \tag{7}$$

$$s_D(r_D,0)=0 \tag{8}$$

$$\lim_{r\to r_w}\frac{1}{2}r_Dq_D=-1+\frac{r_{cD}^2}{4S}\frac{\partial s_{wD}}{\partial t_D} \tag{9}$$

Table 1 Transform Equations in dimensionless

$r_D=\frac{r}{b}$	$q_D=\frac{4\pi b^2}{Q}q$
$r_{cD}=\frac{r_c}{b}$	$s_D=\frac{4\pi K^{\frac{1}{n}}b}{Q}s$
$r_{wD}=\frac{r_w}{b}$	$s_{wD}=\frac{4\pi K^{\frac{1}{n}}b}{Q}s_w$
$r_{iD}=\frac{r_i}{b}$	$t_D=\frac{K^{\frac{1}{n}}t}{Sb}$
$k_D=\frac{4\pi K^{\frac{1}{n}}b^2}{Q}$	

The Izbash's equation can be rewritten with the dimensionless formats as

$$q_D{}^n=k_D^{n-1}\frac{\partial s_D}{\partial r_D} \tag{10}$$

Based on Equation (10), the derivative of q_D in term of r_D is

$$\frac{\partial q_D}{\partial r_D}=\frac{1}{n}k_D^{\frac{n-1}{n}}\frac{\partial s_D}{\partial r_D}^{\frac{1-n}{n}}\frac{\partial^2 s_D}{\partial r_D^2} \tag{11}$$

Substituting Equations (10) and (11) into Equation (6) yields

$$\frac{\partial^2 s_D}{\partial r_D^2}+\frac{n}{r_D}\frac{\partial s_D}{\partial r_D}=nk_D^{\frac{1-n}{n}}\left(\frac{\partial s_D}{\partial r_D}\right)^{\frac{n-1}{n}}\frac{\partial s_D}{\partial t_D} \tag{12}$$

It is known that as time goes on, the flow gradually approaches the "quasi" condition, which implies that the storage release around the pumping well has nearly been completed. Therefore, the flow rate per unit time of any radial surface near the pumping well has been assumed as Q, which can be expressed as $2\pi rbq\approx -Q$. Then to linearize Equation (12), the equations above and an approximation at a late time are given as:

$$\frac{1}{2}r_Dq_D\approx -1 \tag{13}$$

$$\frac{\partial s_D}{\partial r_D}=\frac{(q_D)^n}{K_D^{n-1}}\approx -\frac{\left(\frac{2}{r_D}\right)^n}{K_D^{n-1}} \tag{14}$$

At a late time or near the pumping well, the error introduced by such approximation is proved to be small (Xiao et al. 2019). Combining Equation (14) and Equation (12), the governing equation can be simplified as

$$\frac{\partial^2 s_D}{\partial r_D^2}+\frac{n}{r_D}\frac{\partial s_D}{\partial r_D}=\epsilon r_D^{1-n}\frac{\partial s_D}{\partial t_D} \tag{15}$$

Where: $\epsilon=n\left(\frac{k_D}{2}\right)^{1-n}$.

Substituting Equations (14) and (10) into Equation (9), the boundary condition with the pumping wellbore becomes

$$\lim_{r\to r_w}\frac{1}{2} r_D k_D^{n-1}\frac{\partial s_D}{\partial r_D}\left(\frac{2}{r_D}\right)^{1-n} = -1+\frac{r_{cD}^2}{4S}\frac{\partial s_{wD}}{\partial t_D} \quad (16)$$

According to the Boltzmann transform, this analytical equation for non-Darcian flow with wellbore storage effect is given as follows:

$$s_D = \frac{1}{\left[\left(\frac{2}{k_D r_D}\right)^{1-n}+\frac{r_D^{n-1}}{r_{wD}^{n-1}}\frac{r_{cD}^2}{4S} t_D^{(-1)}\right]\exp\left(-\frac{\epsilon r_{wD}^{1-n}}{4}\left(r_{wD} t_D^{-\frac{1}{2}}\right)^2\right)} \int_{\frac{n r_D^{3-n}}{4t_D}\left(\frac{k_D}{2}\right)^{1-n}}^{\infty}\frac{\exp(-y)}{y}dy\cdots \quad (17)$$

The derivation process of Equation (17) is provided in the supporting information B. Assuming there is no wellbore storage with $r_{cD}=0$, $r_{wD}=0$, Equation (11) is reduced as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1}\int_{\frac{n r_D^{3-n}}{4t_D}\left(\frac{k_D}{2}\right)^{1-n}}^{\infty}\frac{\exp(-y)}{y}dy \quad (18)$$

Then, a general function for drawdown induced by constant rate test under non-Darcian condition is given as follows:

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} MW(u) \quad (19)$$

Where: M is defined as wellbore storage coefficient, it is 1 without wellbore storage and $\frac{1}{\left[1+\frac{r_{cD}^2}{4S r_{wD}^{n-1}} t_D^{(-1)}\left(\frac{2}{k_D}\right)^{n-1}\right]\exp\left(-\frac{\epsilon r_{wD}^{3-n}}{4} t_D^{-1}\right)}$ with wellbore storage; $u=\frac{n r_D^{3-n}}{4t_D}\left(\frac{k_D}{2}\right)^{1-n}$; $W(u)$ is defined as well function.

1.3 Analytical solutions for bounded aquifers

In this section, based on the image-well system with superposition principle, a set of equations to calculate drawdown, for aquifers with one or more straight recharge and barrier boundaries, have been proposed.

According to Stallman work (Kruseman and Ridderna, 1990), it is assumed that the flow is transient and the confined aquifer has six types of boundary combination, which is shown in Fig. 2. In this simulation area, the straight recharge or barrier boundaries fully penetrate the aquifer. The hydraulic head at the recharge boundary is constant and the permeability at the contact surface is the same as that of the aquifer (Lu et al. 2015). The imaginary well system proposed by Stallman and Brown (1951) has been verified to be the most useful method for approximating the mechanism in the bounded aquifer (Kruseman and Ridderna, 1990). Hence, they are applied as shown in Fig. 2 in this paper. The imaginary recharge or injection well works simultaneously with the real well, and injects water to the aquifer or pumps water from the aquifer with the same injecting or withdrawing rate by the real well. Suppose that the dimensionless distance from the real well to the piezometer is r_D, and from the i-imaginary well to the piezometer is r_{iD}. Their ratio is $r_{riD}=r_{iD}/r_D$, an expression is given below

$$u_i = \frac{n r_{iD}^{3-n}}{4t_D}\left(\frac{k_D}{2}\right)^{1-n} = r_{riD}^{3-n}u \quad (20)$$

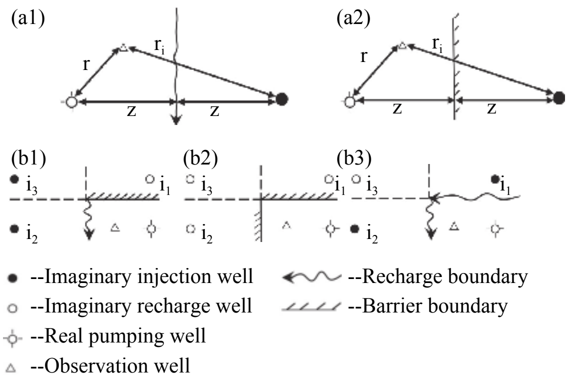

Fig. 2 Schematic sketch of a bounded aquifer with A1) one recharge, A2) one barrier boundary, B1) one barrier and one recharge boundary, B2) two barrier boundaries and B3) two recharge boundaries

The drawdown in the observation well in an arbitrary boundary aquifer can be described as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u)\pm W\left(r_{r1D}^{3-n}u\right)\pm W\left(r_{r2D}^{3-n}u\right)\pm\cdots \pm W\left(r_{rnD}^{3-n}u\right)\right] \quad (21)$$

In the system with one boundary, the drawdown of the one barrier boundary system is depicted as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u)+W\left(r_{r1D}^{3-n}u\right)\right] \quad (22)$$

The drawdown of the one recharge boundary system is expressed as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u)-W\left(r_{r1D}^{3-n}u\right)\right] \quad (23)$$

Equations (22) and (23) can be used when the pumping well is close to a relatively straight river, cliff, coast, vertical impermeable layer and so on.

Otherwise, in the system with two straight boundaries, the drawdown of two boundaries at right angles to each other is explained as:

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u) + W\left(r_{r1D}^{3-n}u\right) + W\left(r_{r2D}^{3-n}u\right) + W\left(r_{rnD}^{3-n}u\right)\right] \quad (24)$$

The drawdown for two straight recharge boundaries arranged at right angles is expressed as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u) - W\left(r_{r1D}^{3-n}u\right) - W\left(r_{r2D}^{3-n}u\right) + W\left(r_{rnD}^{3-n}u\right)\right] \quad (25)$$

For one barrier boundary and one recharge boundary at right angles to each other, the drawdown is assessed as

$$s_D = \left(\frac{2}{r_D k_D}\right)^{n-1} M\left[W(u) + W\left(r_{r1D}^{3-n}u\right) - W\left(r_{r2D}^{3-n}u\right) - W\left(r_{rnD}^{3-n}u\right)\right] \quad (26)$$

Equations (24), (25) and (26) can be used when the pumping well is close to two boundaries that intersect at approximately right angles, for example at the convex-bank of a river, the turn of a coast, or an intersection of the stream and vertical impermeable bedrock.

For two parallel, three and four straight boundaries, the equations of drawdown can also be obtained by using the above method.

2 Verification and discussion

The effectiveness of the proposed solutions is verified by comparison with results from numerical solutions via hypothetical cases. Different from the analytical solutions based on Darcian's law, there are two prime effects involved in the proposed solutions. They are non-Darcian and wellbore storage effects, which is especially discussed in the section.

With a developed MATLAB program, the drawdown-time curve can be easily simulated in aquifers with different boundaries by using Equations (22)-(26). The aquifers with one barrier boundary and with two recharge boundaries at right angles are taken as examples, as shown in Fig. 3, which will be simulated by using Equations (22) and (25), respectively. The presented hypothetical cases are all in-situ pumping tests where the confined aquifer is mainly composed by the mixture of sandstone and gravel. According to Kruseman and Ridderna (1990), the storativity and hydraulic conductivity in these cases should be in the range of $5\times10^{-3} - 5\times10^{-5}$ m/d and $1 - 10^{-3}$ m/d, respectively. To meet the above requirements, the parameters of the six hypothetical cases are listed in Table 2. They have been transformed into dimensionless parameters to ensure that s_D-t_D curves can be obtained by the proposed solution.

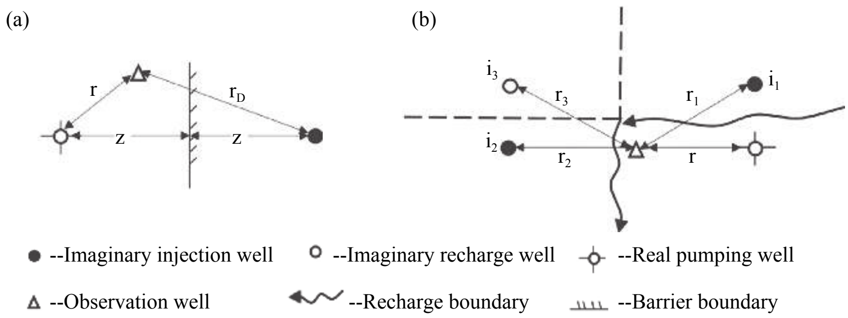

Fig. 3 Schematic sketch of a bounded aquifer with a) one barrier boundary and b) two recharge boundaries at right angles with parameters

Table 2 Parameter values in dimensionless format

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
r_D	0.1	0.02	0.02	0.2	0.2	0.2
r_{wD}	0.02	0.02	0.02	0	0.02	0, 0, 0.02
r_{cD}	0.1	0.05	0.05	0	0.03	0, 0, 0.05
k_D	0.2	0.1, 1, 10	0.2	0.1, 1, 10	0.1, 1, 10	0.2
S	0.001	0.001	0.001	0.001	0.001	0.001
n	1.2	1.2	1, 1.3, 1.5	1.2	1.2	1, 1.5, 1.5

2.1 Verification

Based on the dimensionless parameters in Table 2, Case 1 aims to investigate the reliability of proposed equations by comparison with a finite-element solution. The numerical solution based on the Izbash' s equation is simulated by using the software entitled COMSOL Multiphysics (COMSOL Inc., Burlington, Massachusetts, USA). It is a multiphysics field simulation software based on finite element methods that can be used in most situations of engineering, manufacturing and scientific research. The software provides coupled multi-physics field and single-physics field modelling capabilities and simulation data management, which is reported to be one of the most powerful tools for approximating porous media flow in the world (Hu and Chen, 2008). The transient solution of the numerical model is performed by the second-order Lagrange discrete method, relying on the built-in general form PDE module in COMSOL. In the modeling process, the surrounding infinity boundary is approximated by a Dirich let boundary condition at 10 000 m from the pumping well, and the aquifer is unitized using an extremely fine triangular mesh. In order to simulate the non-Darcian pumping, the governing equation proposed for numerical solution is simply the combination of Boussinesq Equation and Izbash's equation, i.e. Equation (12). Considering that Boussinesq and Izbash' s equations have been adequately verified in actual pumping tests, the non-Darcian drawdown obtained by the numerical model should be considered reliable (Wen et al. 2011, 2014; Xiao et al. 2019; Li et al. 2020).

Fig. 4a) and Fig. 4b) present the type curves of Case 1 by both analytical and numerical solutions, with the solutions of one barrier boundary and two recharge boundaries at right angles. The result indicates that the dimensionless drawdown-time curves by the proposed solutions is fitted well with the numerical solutions, and there is a certain deviation only in the part where curves have a sharp turning point. The difference may be due to the error of the numerical model that is required to ensure convergence when the slope of the curve changes significantly. Therefore, it should be suggested that the proposed analytical solutions are reliable. Furthermore, comparing Fig. 4a) and Fig. 4b), it can be seen that an aquifer with one barrier boundary has a much larger drawdown than an aquifer with two recharge boundaries at right angles. This means that the effect of the boundary is significant in the pumping test. In addition, although the numerical solution estimates a relatively reliable drawdown, its accuracy is significantly reduced when the slope of the s_D-t_D curve changes abruptly. The analytical model can overcome such limitations, which is suitable for assessments of the aquifer hydraulic parameters. A simple analytical solution should be selected than the numerical ones for characterizing groundwater pumping in the boundary aquifer.

2.2 Effect of the non-Darcian characteristics

To further investigate the non-Darcian characteristics of groundwater pumping, two hypothetical cases in Table 2, Case 2 and Case 3, have been proposed to study the dimensionless quasi-hydraulic conductivity and non-Darcian index, respectively. The simulation results of the drawdown under different quasi-hydraulic conductivities are shown in Fig. 5. It clearly indicates that increasing k_D can enhance drawdown in the early pumping times but decrease it in the late pumping times.

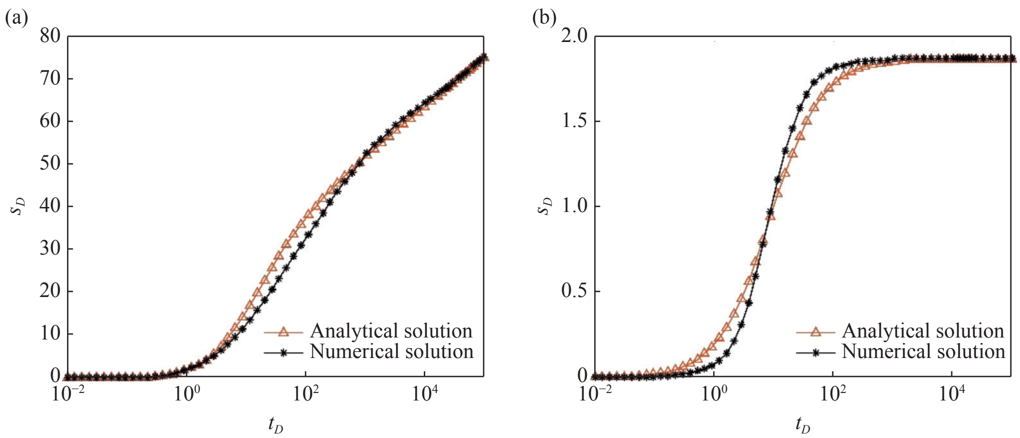

Fig. 4 Comparison with the numerical solution in the hypothetical case with a) one barrier boundary and b) two recharge boundaries at right angles via Case 1

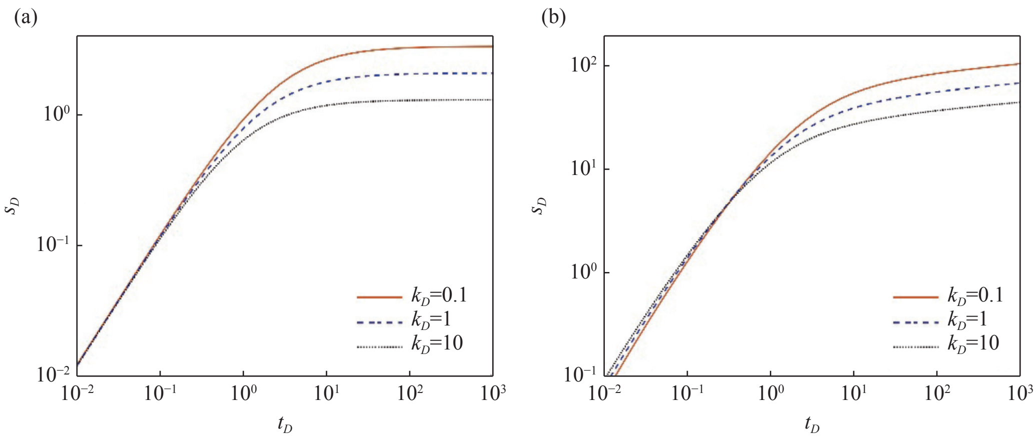

Fig. 5 Type curves with different dimensionless quasi-hydraulic conductivity in wellbore screen with a) two recharge boundaries at right and b) one barrier boundary

The result in Fig. 5 can be explained as follows. A larger k_D implies a more turbulent flow and the pumped water is easier to be transported. At early stage of pumping, the source of flow is mainly the storage of aquifer and a larger k_D improves the release of water, which further causes a larger drawdown. As pumping continues, groundwater in the aquifer located far away from the pumping well gradually become the main source of the outflow. As a result, a more turbulent flow with a larger k_D can lead to a greater groundwater recharge from the further aquifer in the meantime, which helps to slow down the drop of hydraulic head in the observation well even with a larger k_D.

The simulation results obtained from Case 3 are shown in Fig. 6, which is used to investigates the effect of the non-Darcian index. The results have suggested that, with the increase of n, the drawdown increases first and then decrease with the time. Compared to the Darcian flow curves, i.e. the curves for $n = 1$, the drawdown shows a strong deviation. This result proves that the effect of non-Darcian flow is non-negligible. In fact, based on Izbash's equation, an increase in both K and n can lead to an increase in the degree of turbulence flow, which allows the distant recharge to reach the pumping well faster. Therefore, it is reasonable to regard that the effect of n value is similar to that of k_D.

2.3 Effect of the wellbore storage

In order to clarify the influence of wellbore storage, the drawdown-time curves with different quasi-hydraulic conductivity k_D in Case 4 and Case 5 are simulated, as shown in Fig. 7. It is clear that the wellbore storage can result in a deviation of the drawdown-time curve from that without wellbore storage at the early time. Such derivation is increased with k_D value. As pumping continues, the wellbore storage effect is weakened and then completely disappeared at a late time. It is because that, at the early times, the water can be extracted from

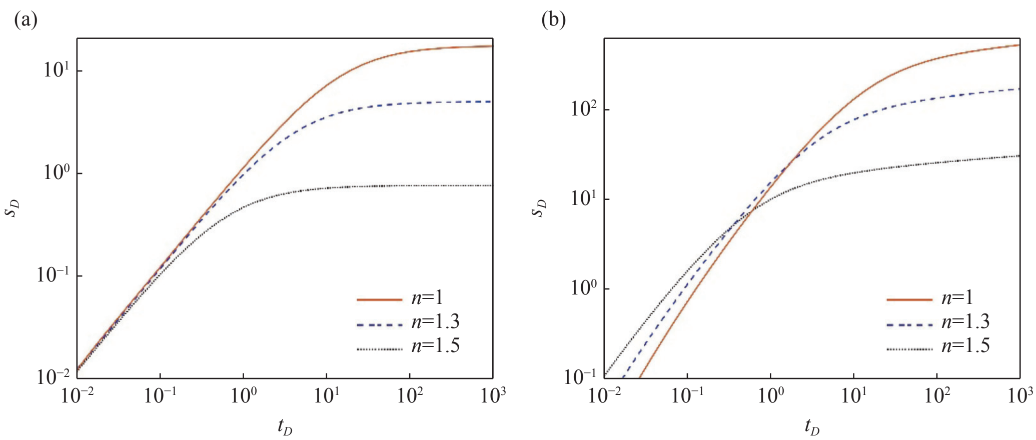

Fig. 6 Type curves with different non-Darcian index in wellbore screen with a) two recharge boundaries at right and b) one barrier boundary

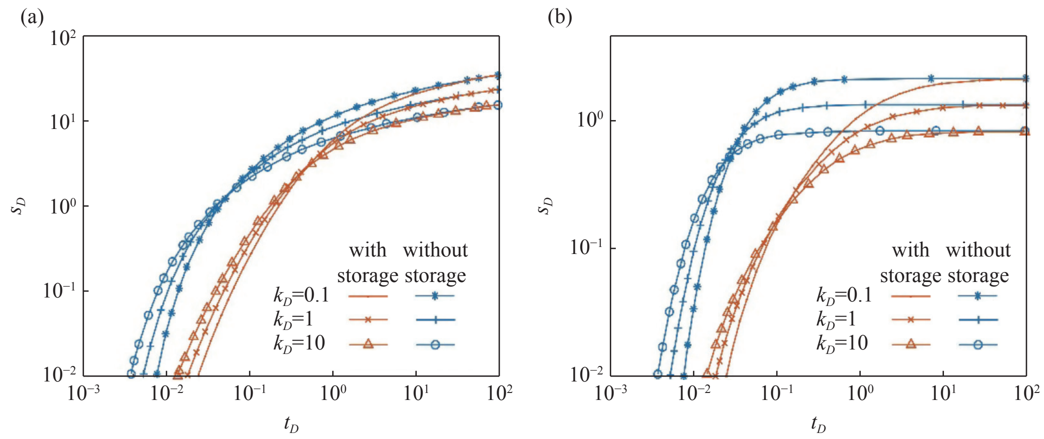

Fig. 7 Type curves with different dimensionless quasi-hydraulic conductivity without and with wellbore storage in an aquifer with a) one barrier boundary and b) two recharge boundaries at right

the wellbore storage. At the late time of pumping, all the water stored in the wellbore is completely released, thus the wellbore storage effect is disappeared. In general, the effect caused by the wellbore storage is mainly in the range of $t_D < 10$, which normally disappears after around 10 minutes of pumping time.

2.4 Combined effect of the non-Darcian and wellbore storage

This section focuses on the combined effect of non-Darcian characteristics and wellbore storage. Based on the parameter values given in Case 6, the results are shown in Fig. 8 in the following three cases: 1) the non-Darcian effect and wellbore storage are not considered (labelled Curve without N and W), 2) the non-Darcian is considered but the wellbore storage is not considered (labelled Curve with N and without W), and 3) the non-Darcian and wellbore storage are both considered (labelled Curve with N and W). It is indicated that the drawdown without the effect of non-Darcian and wellbore storage is decreased in the early pumping time and increased at a later stage. Considering the effect of the wellbore storage, the characteristic of the pumping flow is primarily controlled by the degree of non-Darcian rather than the wellbore storage.

3 Conclusions

This paper proposes a new analytical solution to investigate the non-Darcian pumping-induced flow with wellbore storage in bounded confined aquifers. According to the Izbash's equation, the real bounded aquifer system is substituted by an infinite aquifer system with a series of real and imaginary wells. The simplified solutions for the drawdown simulation are derived by using the Boltzmann transform and superposition principle. The proposed analytical model can be used in the bounded aquifer after corresponding adjustments, and the two most commonly used solutions in practical

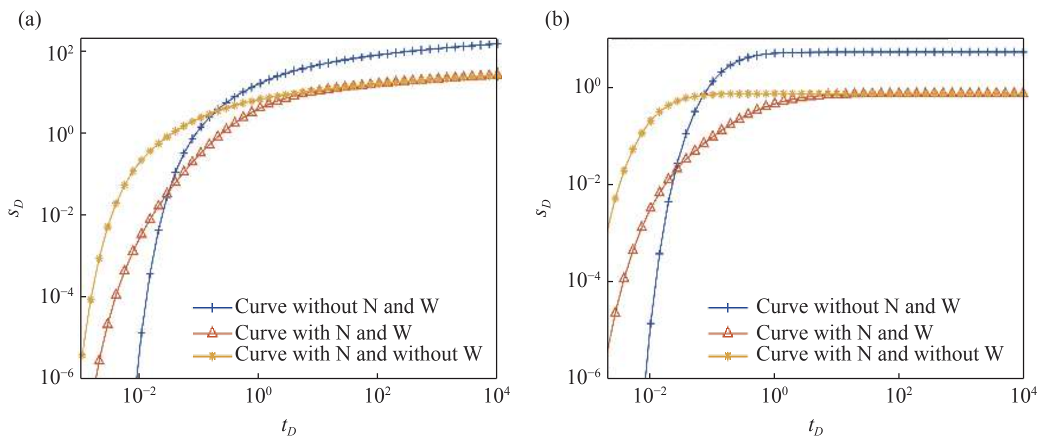

Fig. 8 Type curves are for the investigation of the combined effect of the non-Darcian and wellbore storage in an aquifer with a) one barrier boundary and b) two recharge boundaries at right

tests have been discussed, which are applicable when the pumping well is close to a relatively straight boundary, for example near the river, cliff, coast and vertical impermeable layer, and when two boundaries intersect at nearly right angles, for example at the convex-bank of a river, the turn of a coast, and an intersection of the stream and vertical impermeable bedrock. Further, the effects of the pumping flow influenced by the non-Darcian effect and wellbore storage in different boundary aquifers have been specifically investigated. The main conclusions are highlighted as follows:

(1) The reliability of the analytical solution is proved by comparison with the numerical solution in COMSOL, and it is found that the drawdown in the aquifer with two recharge boundaries is much smaller than that with one barrier boundary.

(2) In the bounded aquifer, non-Darcian effect on drawdown simulation is mathematically affected by the dimensionless quasi-hydraulic conductivity, k_{D}, and the non-Darcian index, n. A greater k_D or n value can cause a larger drawdown at early pumping time but a smaller drawdown at a late time. The non-Darcian degree has a greater effect on pumping flow than the wellbore storage.

(3) Keep other parameters as the same, the wellbore storage can lead to a larger drawdown in the first 10 minutes of pumping, because the pumped water not only comes from the aquifer, but also from the wellbore storage.

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Supporting Information

Additional section can be viewed in the online version of this paper: Supporting Information A shows the detailed clarification associated with the dimensionless transformation. Supporting Information B shows the derivation processes of analytical solutions in bounded confined aquifers with non-Darcian effect.

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