Research on Pisha-sandstone’s anti-erodibility based on grey multi-level comprehensive evaluation method

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Abstract: After a comprehensive analysis for the influential factors like anti-erodibility chemical constitutes, mineral components and micro-structure characteristics of the Pisha-sandstone, and by combining AHP and grey correlation analysis, the anti-erodibility quantitative comparison for 5 types of Pisha-sandstones in Ordos is made on the basis of the grey multi-layer comprehensive assessment mode. The result shows that, from the anti-erodibility point of view, the order of those types is: Pink sandstone, celadon sandstone, purple sandstone, grey sandstone and commixed sandstone. For the evaluation process adopts a simple method, a direct, reasonable and practical result is given, which is also identical to the result of the field survey on soil and rock erosion problems. Consequently, the research on the Pisha-sandstone’s anti-erodibility issue is turned from the qualitative description process to the quantitative evaluation stage.

Keywords: Grey comprehensive evaluation; Multi-layer; Relation; Anti-erodibility; Pisha-sandstone

Introduction

Erdos to the north of the Loess Plateau suffers from the serious rock-soil erosion. In the exposed bedrock region, the erosion modulus (SHI Jian-sheng et al. 2007) can be more than 12 500 t/(km²·a) and even almost as high as 30 000 t/(km²·a). So this region is known as “the world’s top region (ZHANG De-feng and LIU Zhi-gang, 2007)” for soil erosion, and even nicknamed as the “cancer to the earth environment (HAN Xue-shi and SONG Ri-sheng, 1996)” and the “the moon on the earth (MA Chao-de et al. 2006)” . The bedrock that contributes to the serious rock-soil erosion in this region is the “Pisha-sandstone”, one type of sedimentary sandstone. The strong rock-soil erosion phenomenon in the Pisha-sandstone bedrock region is not merely related to the local natural conditions and external driver, but also related to the existence of many erodible Pisha-sandstones in the region.

When an evaluation is made of the sand-soil erosion problems in the Erdos Region, the top concern is to consider the sand-soil ability to prevent the external erosion. The previous researches are mostly focused on how the chemical constituents, physical properties, corrosion and precipitation (crystallization) affect the Pisha-sandstone erodibility (ZHANG Qian et al. 2010; LI Shuai-ying et al. 2008; GU Shi-xiang et al. 1999; WEI Xin-guang et al. 2011). Moreover, the evaluation work for the sandstone’s anti-erosion performance is only limited to the qualitative description, so it is hard to evaluate the modeling needs in the quantitative evaluation of sand-soil erosion. After the internal influence factors of Pisha-sandstone anti-erodibility (SHI Jian-sheng et al. 2007) are subject to qualitative analysis, this paper uses the grey multi-level comprehensive evaluation method (XIAO Xin-ping et al. 2005) to perform the quantitative evaluation of Pisha-sandstone anti-erodibility.
1 Internal cause analysis of Pisha-sandstone anti-erodibility

There are three internal contributors to Pisha-sandstone anti-erodibility, including the chemical constituents, physical properties and micro-structure characteristics of Pisha-sandstone. The following section will give a brief analysis of how each factor affects the Pisha-sandstone’s anti-erosion performance from three perspectives.

1.1 Chemical constituents

The total analysis experiment of Pisha-sandstone sampled silicate indicates (Table 1) that its chemical constituents remains steady, and the steady components like SiO$_2$, Al$_2$O$_3$ and Fe$_2$O$_3$ have the mean content of 65.44%, 11.94% and 2.50%, so the total mean content is more than 79%. It’s true that the unsteady components like CaO, K$_2$O and Na$_2$O have a total content of less than 10%, but their chemical properties prove extremely active. So the chemical alternation will become the frequent occurrences, and the structural damage may often happen to rocks. As a result, rocks will get more vulnerable to the external erosion.

<table>
<thead>
<tr>
<th>Chemical constituents</th>
<th>SiO$_2$</th>
<th>Al$_2$O$_3$</th>
<th>TiO$_2$</th>
<th>Fe$_2$O$_3$</th>
<th>FeO</th>
<th>CaO</th>
<th>MgO</th>
<th>K$_2$O</th>
<th>Na$_2$O</th>
<th>MnO</th>
<th>P$_2$O$_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>65.44</td>
<td>11.94</td>
<td>0.49</td>
<td>2.5</td>
<td>0.8</td>
<td>5.08</td>
<td>2.06</td>
<td>3.28</td>
<td>1.46</td>
<td>0.07</td>
<td>0.16</td>
</tr>
</tbody>
</table>

1.2 Mineral components

The rocks’ anti-weathering performance has much to do with the components and numbers of minerals involved. An attempt to explore the Pisha-sandstones’ anti-erodibility should start with the anti-weathering performance of all the mineral components of such sandstone. The analysis of Pisha-sandstone mineral components (Table 2) shows that the main mineral components include quartz, feldspars and clay minerals.

<table>
<thead>
<tr>
<th>Mineral components</th>
<th>Quartz</th>
<th>Potassium feldspars</th>
<th>Plagioclase</th>
<th>hydromica</th>
<th>Kaolinite</th>
<th>Montmorillonite</th>
<th>Calcite</th>
<th>Muscovite</th>
<th>johnstonite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35.24</td>
<td>20.99</td>
<td>13.54</td>
<td>11.23</td>
<td>4.71</td>
<td>2.75</td>
<td>6.80</td>
<td>2.47</td>
<td>0.34</td>
</tr>
</tbody>
</table>
increasing proportion in rocks, their engineering geological property gets more sensitive to water. For instance, there are some major changes to water affinity, water permeability, malleability and expandability, which can all make rocks more vulnerable to external erosion. For the rock-forming minerals with Pisha-sandstones included, quartz claims quite limited contents, and those poor anti-erosion mineral constituents like feldspar and carbonate minerals can constitute as high as 44% of all the mineral constituents, and clay minerals as high as 19%. This means that the Pisha sandstones are made of many weak anti-erosion rock-forming minerals, and their mineral constituents will have an adverse effect on the rocks’ anti-erosion performance.

1.3 Microstructure characteristics

For rocks, their microstructure characteristics refer to the size, shape, arrangement, porous distribution and inter-particle connections of minerals (or rock debris) (TANG Da-xiong et al. 1999). For Pisha sandstones, the chemical constituents and mineral components are their material basis, and their microstructure conditions are their structural basis. So their microstructure characteristics have an important effect on their anti-erosion performance. The researches on the relation between their microstructure characteristics and their mechanical characteristics (WU Li-jie et al. 2007) show that the main structural features capable of affecting the Pisha-sandstones’ anti-erodibility include grain diameters, standard deviation of grain diameters, shape coefficient, 2D fractal dimension, targeted fractal dimension, 2D fractal dimension of pore diameter and space, as well as the content of substrates. In fact, the size, shape, arrangement and porous distribution of particles mean much to the Pisha-sandstones’ anti-erosion performance. The smaller the minerals’ grain diameter, 2D fractal dimension and targeted fractal dimension are, the higher the Pisha-sandstones’ physical strength and the anti-pressure and tensile strength will become.

2 Grey multi-level comprehensive evaluation method

The qualitative analysis of those internal factors that affect the Pisha-sandstones’ anti-erosibility alone can hardly meet the requirements of quantitative evaluation. To achieve more accurate, comprehensive and scientific exploration into and evaluation of the relation between Pisha-sandstones’ anti-erosibility and rock properties, grey relational analysis method and the analytic hierarchy process should be combined to perform the quantitative evaluation of Pisha-sandstones’ anti-erosibility based on the grey multi-level comprehensive evaluation model. The grey multi-level comprehensive evaluation method can be divided into single-level comprehensive evaluation method and multi-level comprehensive method (XIAO Xin-ping et al. 2005).

2.1 Single-level comprehensive evaluation method

The single-level comprehensive evaluation method mainly involves the steps like the determination of evaluation indices, determination of optimal index set, normative data processing, determination of evaluation matrix, determination of different weights and grey comprehensive evaluation.

(1) Determination of evaluation indices: Evaluation indices mean different properties or skills of evaluation targets. They’re the evaluation basis for evaluating the related targets.

(2) Determination of optimal index set: Optimal index set is the best dataset of the same indicator for different evaluation targets. It’s the benchmark for comparing different evaluation targets.

Set $C_k(i=1, 2, \ldots, n; k=1, 2, \ldots, m)$ as the original data of the $k^{th}$ indicator for the $i^{th}$ sample in the system concerned. If a matrix is involved, the original data can be expressed as $C=(C_{ik})_{n \times m}$. Specifically, $X$ is a matrix of Row $n$ and Line $m$; set $X^*_k$ as the optimal value of each evaluation sample for the indicator $k$ (the maximum value for optimal indicator and the minimum value for low indicator), so $C^*=(C^*_1, C^*_2, \ldots, C^*_m)^T$ is the optimal index set of this system.

(3) Normative data processing: The indices involved in evaluation process usually have different dimensions and orders of magnitude, so it’s impossible to compare these indices. In this...
case, they should be subject to dimensionless and normalized processing, namely the normative processing. In the relational analysis, the cost-based and benefit-based indices should have their parameters subject to the normative processing according to the following formula (1, 2):

\[
c_i(k) = \frac{c_i(k) - c_{i\min}}{c_{i\max} - c_{i\min}}, \quad (i = 1, 2, ..., n; k = 1, 2, ..., m) \quad (1)
\]

\[
c_i(k) = \frac{c_i(k) - c_{i\min}}{c_{i\max} - c_{i\min}}, \quad (i = 1, 2, ..., n; k = 1, 2, ..., m) \quad (2)
\]

(4) Determination of evaluation matrix: With \( \mathbf{C} = [C_{11}, C_{12}, ..., C_{im}] \) as the reference data series, \( \mathbf{C} = [C_{11}, C_{12}, ..., C_{im}] \) can be used as the comparable data series. Use the formula 3:

\[
\xi_i(k) = \min \left\{ \frac{|c_i(k) - c_{i\min}|}{c_{i\max} - c_{i\min}}, \max \left\{ \frac{|c_i(k) - c_{i\max}|}{c_{i\max} - c_{i\min}} \right\} \right\} + \rho \max \left\{ \frac{|c_i(k) - c_{i\min}|}{c_{i\max} - c_{i\min}} \right\}
\]

In the formula, \( \min |c_i(k) - c_{i\min}| \) is the two levels of minimum differential, and \( \max |c_i(k) - c_{i\min}| \) is the two levels of maximum differential.

\( \rho \) is the identification coefficient which the value range of [0,1]. In most cases, \( \rho = 0.5 \).

The relational coefficient \( \xi_i(k) \) for the \( k \)th indicator and the \( k \)th optimal indicator of the \( i \)th sample is obtained, so the evaluation matrix can be described as \( \mathbf{E}(\xi_i(k))_{n \times m} \).

(5) Determination of different weights: All the evaluation indices may affect the evaluate targets differently. To reflect the actual conditions, the dualistic comparison method can determine \( W \), the weighting coefficient matrix of all the evaluation indices. \( W = (w_1, w_2, ..., w_m) \) means the weighting distribution matrix of \( m \) evaluation indices, where \( W_k(k = 1, 2, ..., m) \), as the weighting of the \( k \)th evaluation indicator, should meet the following condition:

\[
\sum_{k=1}^{m} w_k = 1 \quad (4)
\]

(6) Gray comprehensive evaluation: Based on the evaluation matrix and weighting matrix, the comprehensive evaluation result matrix can be expressed in the form of gray relation:

\[
\mathbf{R} = \mathbf{W} \cdot \mathbf{E}(r_1, r_2, ..., r_n)
\]

where, \( r_i = \sum_{k=1}^{m} w_k \xi_i(k) \) is the relation. In the model, \( \mathbf{R} = \mathbf{W} \cdot \mathbf{E}(r_1, r_2, ..., r_n) \) means the comprehensive evaluation result matrix of \( n \) evaluation samples. If the relation \( r_i \) is the largest, the \( i \)th evaluation sample is better than other samples. In this way, the order of magnitude can be arranged for all the samples.

2.2 Grey multi-level comprehensive evaluation method

When the indices for evaluation targets differ in their hierarchical structure, it’s imperative to perform the multi-level comprehensive evaluation. The multi-level evaluation is based on the single-level comprehensive evaluation. Two evaluation methods are similar, but have different data processing performances. The first-level data depend exclusively on the second-level data, and the dimensionless data processing is not necessary. This is the same as the fuzzy comprehensive evaluation. The second-level evaluation can be directly used to create the first-level evaluation matrix. Use the formula (5) to calculate the first-level calculation results.

3 Grey multi-level comprehensive evaluation of Pisha-sandstones’ anti-erodibility

3.1 Evaluation target and purpose

In the Pisha-sandstones’ bedrock distribution area of the Erdos Basin, the most commonly seen Pisha sandstones can be roughly divided into five major types, including gray sandstones, purple sandstones, pink sandstones, celadon sandstones and commixed sandstone. The evaluation samples of this model are five types of sandstones as mentioned above. The anti-erodibility of these sandstones can be subject to the quantitative evaluation from three perspectives including chemical constituents, mineral components and microstructure characteristics.

3.2 Comprehensive evaluation index system

The Pisha-sandstones’ anti-erodibility evaluation system consists of two levels. The first level involves chemical constituents, mineral components and microstructure characteristics, and the second level includes specific evaluation factors of
all factors in chemical constituents, mineral components and microstructure characteristics.

Set $C_i$ as the evaluation index system, $i=1$, 2, 3 represents three evaluation factors including “chemical constituents”, “mineral components” and “parameters of microstructure characteristics”. so $C_1=(c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16})$; $C_2=(c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{27})$; $C_3=(c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}, c_{38})$. $c_{11}$-$c_{16}$ represent the contents of SiO$_2$, Al$_2$O$_3$, Fe$_2$O$_3$, CaO, K$_2$O and Na$_2$O separately; $c_{21}$-$c_{27}$ represent the contents of quartz, potassium feldspar, plagioclase feldspar, carbonate salt, hydromica, kaolinite and montmorillonite representatively; $c_{31}$-$c_{38}$ represent the average grain diameter, standard deviation of grain diameters, shape coefficient, 2D fractal dimension, 2D targeted fractal dimension, average pore diameter, 2D distribution fractal dimension and content of interstitial matter.

### 3.3 Determination of evaluation index weight

With no specific focus, the comprehensive evaluation will prove meaningless. It’s necessary to determine the weights for all the indices. In the Pisha-sandstones’ anti-erodibility evaluation system, the dualistic comparison method should be used to determine the weights for all the indices at all levels.

The first-level weight: $W=(w_1, w_2, w_3)=(0.105, 0.637, 0.258)$.

The second-level weight: $W_i=(w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16})=(0.620, 0.202, 0.052, 0.042, 0.042, 0.042)$; $W_2=(w_{21}, w_{22}, w_{23}, w_{24}, w_{25}, w_{26}, w_{27})=(0.610, 0.123, 0.076, 0.047, 0.048, 0.048, 0.048)$; $W_3=(w_{31}, w_{32}, w_{33}, w_{34}, w_{35}, w_{36}, w_{37}, w_{38})=(0.145, 0.355, 0.042, 0.066, 0.066, 0.151, 0.075, 0.100)$.

### 3.4 Gray multi-level comprehensive evaluation

#### 3.4.1 Single-level comprehensive evaluation

For the chemical constituents (%), mineral components (%) and microstructure characteristics of Pisha-sandstones’ anti-erodibility, the original evaluation data matrix like $C_1$, $C_2$ and $C_3$ are given as follows:

$$C_1 = \begin{bmatrix}
67.6 & 8.50 & 1.0 & 7.9 & 3.2 & 0.7 \\
66.7 & 11.6 & 1.7 & 5.9 & 3.4 & 2.0 \\
67.3 & 0.11 & 2.3 & 5.3 & 3.5 & 1.4 \\
72.4 & 8.7 & 0.8 & 5.5 & 3.4 & 0.9 \\
68.7 & 12.1 & 1.3 & 4.1 & 3.5 & 1.6
\end{bmatrix}$$

$$C_2 = \begin{bmatrix}
37.195 & 24.38 & 4.745 & 25.995 & 2.615 & 1.68 & 0.56 \\
38.107 & 26.093 & 18.007 & 4.54 & 4.297 & 3.453 & 1.97 \\
50.613 & 26.348 & 9.358 & 3.503 & 6.783 & 1.473 & 0.85 \\
36.52 & 26.2 & 22.56 & 11.09 & 0.96 & 0.38 & 0.19
\end{bmatrix}$$

And their optimal index sets including $C_1^*$, $C_2^*$ and $C_3^*$ are given as below:

$C_1^*=(72.4, 12.1, 2.3, 4.1, 3.2, 0.7)$

$C_2^*=(50.613, 26.345, 22.56, 3.502, 0.96, 0.38, 0.19)$

$C_3^*=(3.762, 1.02, 0.796, 1.681, 0.833, 3.564, 1.636, 0.428)$

The known data are used to normalize the data according to the formula (1) or (2), and after the formula (3) and (5) are substituted, the gray relation $R_1$, $R_2$ and $R_3$ of chemical constituents, mineral components and microstructure characteristics can be described as follows separately:

$$R_1=(0.415, 0.447, 0.460, 0.782, 0.568)$$

$$R_2=(0.396, 0.478, 0.493, 0.888, 0.632)$$

$$R_3=(0.419, 0.448, 0.625, 0.790, 0.621)$$

#### 3.4.2 Multi-level comprehensive evaluation

The gray single-level comprehensive evaluation results of the chemical constituents, mineral components and microstructure characteristics can be considered as $C$, the original index data matrix of the high-level comprehensive evaluation process. Specifically, $C=(R_1, R_2, R_3)^T$. The calculation produces the optimal index set $C^*=(0.782, 0.888, 0.632)$. 

http://gwse.iheg.org.cn
The gray relational matrix of the comprehensive factors like the chemical constituents, mineral components and microstructure characteristics can be produced according to Formula (3), and together with the weight \(W=(0.105, 0.637, 0.258)\), then Formula (5) is used to obtain \(R\), all the evaluation targets' gray relation with the comprehensive factors like the chemical constituents, mineral components and microstructure characteristics. Specifically, \(R=(0.357, 0.390, 0.443, 1.000, 0.482)\).

4 Analysis and discussion of evaluation results

4.1 Analysis of single-level evaluation results

(1) Evaluation result based on chemical constituents

We learn from \(R_1\) that \(r_{14} > r_{15} > r_{13} > r_{12} > r_{11}\). Five types of Pisha-sandstones can be arranged in the order of anti-erosion performance: pink sandstones, celadon sandstones, purple sandstones, gray sandstones, and commixed sandstone.

(2) Evaluation result based on mineral components

We learn from \(R_2\) that \(r_{24} > r_{25} > r_{23} > r_{22} > r_{21}\). Five types of Pisha-sandstones can be arranged in the order of anti-erosion performance: Pink sandstones, celadon sandstones, purple sandstones, gray sandstones, and commixed sandstone.

(3) Evaluation result based on microstructure characteristics

We learn from \(R_3\) that \(r_{34} > r_{33} > r_{35} > r_{32} > r_{31}\). Five types of Pisha-sandstones can be arranged in the order of anti-erosion performance: Pink sandstones, purple sandstones, celadon sandstones, gray sandstones, and commixed sandstone.

The single-level evaluation results can be used to analyze a single factor’s contribution to different Pisha-sandstones’ anti-erosion performance and to identify their anti-erodibility difference. For instance, when we evaluate the celadon sandstones’ anti-erosion performance, we’ll find that the mineral components contribute to their anti-erodibility than microstructure characteristics.

4.2 Analysis of multi-level evaluation results

The comprehensive evaluation (\(R\)) of three factors like the chemical constituents, mineral components and microstructure characteristics reveals \(r_4 > r_5 > r_3 > r_2 > r_1\). Five types of Pisha-sandstones can be arranged in the order of anti-erosion performance: Pink sandstones, celadon sandstones, purple sandstones, gray sandstones, and commixed sandstone. This evaluation result is consistent with the field observation. For instance, in the outdoor survey on rock erosion, the sandstones with overlapped layers suffer the most serious weathering process. The gray sandstones tend to become the loose sediments.

5 Conclusions

The Pisha-sandstones’ anti-erodibility evaluation is an extremely complicated project. The analysis of Pisha-sandstones’ anti-erosion in the previous surveys on Pisha-sandstones’ erosion problems is limited only to the qualitative descriptions, so can hardly meet the requirements for quantitative evaluations. Given the fact that many factors are included in the Pisha-sandstones’ anti-erosibility evaluation system, including known and unknown information, these factors are mutually affected and can hardly be differentiated from each other. This paper uses the gray multi-level comprehensive evaluation model, which combines the analytic hierarchy process and gray relational analysis, to effectively utilize the grayness and fuzziness of evaluation index and guarantee the simple evaluation process and the visible, comparable and pragmatic evaluation results. This result is consistent with the field survey result of Pisha-sandstones’ erosion problems. Meanwhile, the survey on Pisha-sandstones’ anti-erodibility issue turns qualitative elaboration towards the quantitative evaluation.

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